Reports in Mechanical Engineering
Vol. 3, No. 1, 2022, pp. 210-226
ISSN: 2683-5894, DOI: https://doi.org/10.31181/rme20020042022a

Interval-Valued Picture Fuzzy Maclaurin Symmetric Mean Operator with application in Multiple Attribute Decision-Making

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Article Info

Article history:

Received March 15, 2022
Revised April 18, 2022
Accepted April 20, 2022

Keywords:

Maclaurin Symmetric Mean,
Picture Fuzzy Set,
Aggregation Tools,
Decision Making.

ABSTRACT

A lot of fuzzy models have been planned and researched to review the information under uncertainty and ambiguity. Among these, the model of the interval-valued picture fuzzy set (IVPFS) is very important which can explain the information by four possibilities in the opinion of experts using a membership degree (MD), non-membership degree (NMD), abstinence degree (AD), and a refusal degree (RD) in the form of intervals. The gathering of data is difficult all the time, particularly when the difference of opinions is connected. Due to these four degrees, the use of IVPFS is the best technique to obtain the maximum information while gathering the data from the phenomenon of the real life. This article aims to explore the idea of a Maclaurin symmetric mean (MSM) operator in the framework of IVPFS. In this article, we have studied MSM in the framework of IVPFSs and discussed their application in picking the most suitable company benefit plan (CBP) using interval-valued picture fuzzy (IVPF) data. The proposed operators IVPF MSM (IVPFMSM), IVPF weighted MSM (IVPFWMSM), IVPF dual MSM (IVPFDMSM), and IVPF dual weighted MSM (IVPFDWMSM) operators are found trustworthy with the basic properties. Finally, to show the importance and significance of proposed method, a numerical example has been provided and results have been compared with some existing operators.

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1. Introduction

The notion of the fuzzy set (FS) was introduced by Zadeh (1965) to deal with uncertain information with the help of MD having a value from $[0,1]$. After that Atanassov (1986) improved this concept of FS by giving the idea of the intuitionistic fuzzy set (IFS), in which he used the concept of NMD additionally. IFS is important due to the NMD, which indicated how much an element is not related to a phenomenon. He also expanded the concept of IFS (Atanassov, 1989) into interval-valued IFS (IVIFS) to involve more information precisely. The issue with the IFS was that some of the objects from $[0,1]$ could not be expressed as IFS. To extend the range of IFS, the idea of Pythagorean FS (PyFS) was given by Yager (2013) in which he used the condition that the sum of the square of MD and NMD should be in $[0,1]$. Peng and Yang (2013) expanded the concept of PyFS into IVPyFS to describe the information more precisely. To further increase the range of MD and NMD, Yager (2016) gave the idea of the q-Rung Orthopair Fuzzy Set. The frameworks mentioned above had only two degrees MD and NMD to describe the real-life phenomenon. Cuong et al. (2013) gave the idea of PFS with the addition of an abstinence degree (AD). A PFS is a more important framework due to the involvement of AD which increased the range of accuracy of obtained information from real-life situations. To involve more information some interesting literature can be found in (Ghaznavi et al., 2016) parametric equations and (Jafari et al., 2021) differential equations.

Aggregation operators (AOs) have always been useful tools for gathering information due to which they are very applicable in all fields of sciences such as (Biswas et al., 2021; Wang et al., 2018; Milosevic et al., 2021; Pamucar and Savin, 2020; Pamucar and Dimitrijevic, 2021; Hussain et al., 2022) proposed some families of AOs and applied them into the decision-making problems. These AOs are very important due to their applications in distinct domains like decision making, the measure of the continuous domain (kouloumpis, 2008), the temperature control system of the thermoforming machine (Singhala, 2014), and medical diagnosis. (Adlasing and Mehmood et al., 2021) also developed AOs and applied them to the decision-making problems. (Ali et al., 2021) developed AOs and applied them to decision-making problems. Garg (2017) developed some AOs for the framework of PFS and applied them to decision-making. Looking at the priorities in the initial information, important AOs developed by Yu and Xu (2013), Liu et al., (2020) and Mehmood et al., (2021) have been used in FS theory and have vast application. Many authors have been working on the aggregation operators for PFS during the last decades.

Many AOs have been developed such as arithmetic mean operator, geometric mean operator, Bonferroni mean operator, Heronian mean operator, etc. (Coung et al., 2013) discovered the concept of PFS and designed AOs on it and applied it to decision-making problems. Manemaran and Nagarajan (2006) have generated the concept of the Temporal N Picture fuzzy soft domain. Wang and Zhou (2011) have developed the concept of geometric AOs and their applications in multiple attribute decision-making. Many prioritizations among the input information have been analyzed in many fuzzy frameworks, such as Dombi (1982) arguments. Due to this, some important operators are being introduced such as Heronian mean (HM) operators (Lin et al., 2022) Hammy mean operators (Garg et al., 2021), and arithmetic mean operators (Wei et al., 2010).

Maclaurin symmetric mean (MSM) (Maclaurin, 2013) operators are very important among the methods of aggregation. Unlike the traditional AOs, MSM AOs relate more than two input arguments. So, MSM operators are more significant and elastic. Qin et al. (2015) developed the idea of intuitionistic fuzzy MSM operators (IFMSM) and more worked by Liu et al. (2020) for multi-attribute decision making (MADM) on IFMSM operators. Wei et al. (2019) and Pang et al. (2011) worked on Pythagorean fuzzy MSM (PyFMSM) operators and their applications in decision making. Wei et al. (2019) and Liu et al. (2018) introduced the concept of q-rung orthopair fuzzy MSM (qROFMSM) and applied it in decision making. Qin et al. (2015) developed the concept of MSM operators in a hesitant fuzzy set. Ullah (2021) developed the idea of MSM operators for PFS and used them in decision-making. In this research, we have worked on the IVPFMSM operator, which increased the accuracy of PFS due to the MSM operator. It is the most important and widely operator due to the use of AD and RD.

A IVPFS can express an uncertain situation by using MD, NMD, AD, and RD and they are significant in MADM, pattern recognition, clustering, and medical diagnosis. As we discussed above that traditionally AOs gave the information about the weight vector and cannot relates more than two input information that creating a loss in information under the uncertain situation. The advantage of MSM operators is to correlate the input arguments in the aggregation process under uncertain conditions. So, MSM operators are more important and elastic than traditionally operators such as arithmetic mean operators, geometric mean operators, Dombi t-norm and t-conorm operators, Hamacher t-norm and t-conorm operators, etc. In IVPFS, we used four possible aspects MD, NMD, AD, and RD under the uncertain situation that reduced the loss of information in the decision-making approach. We are motivated to improve the results due to the facts mentioned above. In this research, we aim to develop the concept of MSM operators on IVPFSs. IVPFMSM operators can aggregate more than two input differences of opinion in information science eventually. IVPFMSM gave us more significant and elastic information and eliminate bereavement. The summary of this paper is given below:

1. First, we developed some operations of IVPFS to define the IVPFMSM operator and then explain some basic properties of the IVPFMSM operator (Idempotency, Boundedness, monotonicity).
2. After that, we use the process of multi attributes decision making (MADM) to show the importance of the IVPFMSM operator. We also discuss an example (Company Benefit Plan) mathematically using the MADM process.
3. In the end, we explain the uniqueness of the proposed operator (IVPFMSM) and took a comparative analysis of it with other developed operators. We show that the IVPFMSM operator is more effective than other operators.

In this article, we recall some important definitions in section 2. In section 3, we develop the concept of IVPFMSM operators and IVPFWMSM operators. In section 3, we also develop the concept of interval-valued picture fuzzy dual MSM (IVPFDMSM) and an interval-valued weighted picture fuzzy dual MSM

(IVWPFDMSM) operators. In section 4, we proved that IVPFMSM operators are more beneficial than MSM operators. In part 5, a comprehensive numerical example is elaborated using the MADM application where the Company Benefit Plan (CBP) is described. We analyzed the comparative study of suggested operators mathematically in section 6. Section 7 consists of some conclusive remarks.

2. Preliminaries

This section recalls some basic concepts regarding this article.

Definition 1: On a set X, an IVPFS is of the shape. $H = \{(\tilde{e}, ([a^l, a^u], [b^l, b^u], [d^l, d^u])): 0 \leq sum(a^u, b^u, d^u) \leq 1\}$. Further, $r(\tilde{e}) = [r^l, r^u] = \left[1 - sum(a^l(\tilde{e}), s^l(\tilde{e}), d^l(\tilde{e})), 1 - sum(a^u(\tilde{e}), s^u(\tilde{e}), d^u(\tilde{e}))\right]$ represents the RG of $\tilde{e} \in X$ and the triplet $([a^l, a^u], [b^l, b^u], [d^l, d^u])$ is as an interval-valued PFN (IVPFN).

Definition 2: The basic operations on IVPFs are defined as

Let $H_1 = \{(\tilde{e}, [a_1^l, a_1^u], [b_1^l, b_1^u], [d_1^l, d_1^u]): 0 \leq sum(a_1^u, b_1^u, d_1^u) \leq 1\}$

And

$H_2 = \{(\tilde{e}, [a_2^l, a_2^u], [b_2^l, b_2^u], [d_2^l, d_2^u]): 0 \leq sum(a_2^u, b_2^u, d_2^u) \leq 1\}$ be two IVPFS.

Further,

$$r_1(\tilde{e}) = [r_1^l, r_1^u] = \left[1 - sum\left(a_1^l(\tilde{e}), b_1^l(\tilde{e}), d_1^l(\tilde{e})\right), 1 - sum\left(a_1^u(\tilde{e}), b_1^u(\tilde{e}), d_1^u(\tilde{e})\right)\right]$$

And

$$r_2(\tilde{e}) = [r_2^l, r_2^u] = \left[1 - sum\left(a_2^l(\tilde{e}), b_2^l(\tilde{e}), d_2^l(\tilde{e})\right), 1 - sum\left(a_2^u(\tilde{e}), b_2^u(\tilde{e}), d_2^u(\tilde{e})\right)\right]$$

represents the RG of $\tilde{e} \in X$.

$$H_1 \otimes H_2 = \left\{\left(\begin{array}{c} [a_1^l(\tilde{e}) a_2^l(\tilde{e}), a_1^u(\tilde{e}) a_2^u(\tilde{e})] \\ \begin{bmatrix} 1 - \left(1 - b_1^l(\tilde{e})\right)\left(1 - b_2^l(\tilde{e})\right), \\ 1 - \left(1 - b_1^u(\tilde{e})\right)\left(1 - b_2^u(\tilde{e})\right) \end{bmatrix}, \\ \begin{bmatrix} 1 - \left(1 - d_1^l(\tilde{e})\right)\left(1 - d_2^l(\tilde{e})\right), \\ 1 - \left(1 - d_1^u(\tilde{e})\right)\left(1 - d_2^u(\tilde{e})\right) \end{bmatrix} \end{array}\right), \tilde{e} \in X\right\}$$

$$H_1 \oplus H_2 = \left\{\left(\begin{array}{c} \begin{bmatrix} 1 - \left(1 - a_1^l(\tilde{e})\right)\left(1 - a_2^l(\tilde{e})\right), \\ 1 - \left(1 - a_1^u(\tilde{e})\right)\left(1 - a_2^u(\tilde{e})\right) \end{bmatrix}, \\ [b_1^l(\tilde{e}) b_2^l(\tilde{e}), b_1^u(\tilde{e}) b_2^u(\tilde{e})], \\ [d_1^l(\tilde{e}) d_2^l(\tilde{e}), d_1^u(\tilde{e}) d_2^u(\tilde{e})] \end{array}\right), \tilde{e} \in X\right\}$$

$$\lambda H_1 = \left\{\left(\begin{array}{c} \left[1 - \left(1 - a_1^l(\tilde{e})\right)^\lambda, 1 - \left(1 - a_1^u(\tilde{e})\right)^\lambda\right], \\ \left[\left(b_1^l(\tilde{e})\right)^\lambda, \left(b_1^u(\tilde{e})\right)^\lambda\right], \\ \left[\left(d_1^l(\tilde{e})\right)^\lambda, \left(d_1^u(\tilde{e})\right)^\lambda\right] \end{array}\right), \tilde{e} \in X\right\}$$

$$H_1{}^\lambda = \left\{\left(\begin{array}{c} \left[\left(a_1^l(\tilde{e})\right)^\lambda, \left(a_1^u(\tilde{e})\right)^\lambda\right], \\ \left[1 - \left(1 - b_1^l(\tilde{e})\right)^\lambda, 1 - \left(1 - b_1^u(\tilde{e})\right)^\lambda\right], \\ \left[1 - \left(1 - d_1^l(\tilde{e})\right)^\lambda, 1 - \left(1 - d_1^u(\tilde{e})\right)^\lambda\right] \end{array}\right), \tilde{e} \in X\right\}$$

Definition 3: For the comparison of two IVPFNs $H_1 = \{(\tilde{e}, [a_1^l, a_1^u], [b_1^l, b_1^u], [d_1^l, d_1^u]): 0 \leq sum(a_1^u, b_1^u, d_1^u) \leq 1\}$ and $H_1 = \{(\tilde{e}, [a_1^l, a_1^u], [b_1^l, b_1^u], [d_1^l, d_1^u]): 0 \leq sum(a_1^u, b_1^u, d_1^u) \leq 1\}$, we have the following score function:

$$S(q) = q_{st} = \frac{(a^l)\left(1 - (b^l) - (d^l)\right) + (a^u)\left(1 - (b^u) - (d^u)\right)}{3}$$

Where $S(q) \in [-1,1]$

Definition 4: Let $\alpha_s = (s = 1,2,3, \dots, m)$ be a collection of positive real numbers and $x = (1,2,3, \dots, m)$. Then

$$MSM^{(x)}(a_1,a_2,\dots,a_m)=\left(\frac{\sum \overset{x}{\underset{t=1}{\pi}} a_{st}}{C_m^x}\right)^{\frac{1}{x}}$$

is called MSM. Where $(s_1,s_2,\dots,s_x)$ convert all the x-tuple combination of $(1,2,\dots,m)$, C_m^x is the binomial coefficient.

Definition 5: The MSM have the following properties.

1. $MSM^x(0,0,0,\dots,0)=0;$
2. $MSM^x(a,a,a,\dots,a)=a;$
3. $MSM^x(a_1,a_2,\dots,a_m)\le MSM^x(b_1,b_2,\dots,b_m)$ if all $a_s\le b_s$ for $s=1,2,3,\dots,$

3. Interval valued Picture Fuzzy Maclaurin Symmetric Mean;

In this section, we develop IVPFMSM and IVPFWMSM in the format of IVPFSs. In our forward study, we will mean by w_t=$(w_1,w_2,\dots,w_m)^T$ the weight vector of q_t where $w_t>0$ and $\sum_{t=1}^m w_t=1$

Definition 5: Let $\alpha=([a_s^l,b_s^l,d_s^l],[a_s^u,b_s^u,d_s^u])$, $\beta=([a_t^l,b_t^l,d_t^l],[a_t^u,b_t^u,d_t^u])$ be two IVPFNs. Then, the IVPFMSM operator is given by

$$IVPFMSM\left(q_1,q_{2,\dots,}q_m\right)=\left(\left[\left(\frac{\bigoplus_{1\le s_1\le,\dots,s_x\le m}\left(\bigotimes_{t=1}^{x} q_{st}^l\right)}{C_m^x}\right)^{1/\dot{x}},\left(\frac{\bigoplus_{1\le s_1\le,\dots,s_x}\left(\bigotimes_{t=1}^{x} q_{st}^u\right)}{C_m^x}\right)^{1/\dot{x}}\right]\right) \quad (1)$$

Theorem 1: Let $\alpha=([a_s^l,a_s^u],[b_s^l,b_s^u],[,d_s^l,d_s^u]),\beta=([a_t^l,a_t^u],[b_t^l,b_t^u],[,d_t^l,d_t^u])$ be a collection of IVPFNs. Then, using IVPFMSM operators, we have

$$IVPFMSM(q_1,q_2,\dots,q_m)$$
$$=\left(\begin{array}{c}\left[\begin{array}{c}\left(1-\left(\prod_{\substack{1\le s_1\le,\dots,\\ <s_x\le m}}\left(1-\left(\prod_{t=1}^{\dot{x}} a_{st}^l\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}},\\ \left(1-\left(\prod_{\substack{1\le s_1\le,\dots,\\ <s_x\le m}}\left(1-\left(\prod_{t=1}^{\dot{x}} a_{st}^u\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}\end{array}\right],\left[\begin{array}{c}1-\left(1-\left(\prod_{\substack{1\le s_1\le,\dots,\\ <s_x\le m}}\left(1-\left(\prod_{t=1}^{\dot{x}}(1-b_{st}^l)\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}},\\ 1-\left(1-\left(\prod_{\substack{1\le s_1\le,\dots,\\ <s_x\le m}}\left(1-\left(\prod_{t=1}^{x}(1-b_{st}^u)\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}\end{array}\right]\\ \left[\begin{array}{c}1-\left(1-\left(\prod_{\substack{1\le s_1\le,\dots,\\ <s_x\le m}}\left(1-\left(\prod_{t=1}^{\dot{x}}(1-d_{st}^l)\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}},\\ 1-\left(1-\left(\prod_{\substack{1\le s_1\le,\dots,\\ <s_x\le m}}\left(1-\left(\prod_{t=1}^{\dot{x}}(1-d_{st}^u)\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}\end{array}\right]\end{array}\right) \quad (2)$$

Proof: By using definition 6, we have

$$\bigotimes_{t=1}^{x} q_{st} = \left(\begin{array}{c} \left[\prod_{t=1}^{x} a_{st}^{l}, \prod_{t=1}^{x} a_{st}^{u}\right], \left[1-\prod_{t=1}^{x}(1-b_{st}^{l}), 1-\prod_{t=1}^{x}(1-b_{st}^{u})\right], \\ 1-\left[\prod_{t=1}^{x}(1-b_{st}^{l}), 1-\prod_{t=1}^{x}(1-b_{st}^{u})\right] \end{array} \right),$$

$$\bigoplus_{\substack{1\le s_1\le,\dots,\\ <s_x\le m}} \left(\bigotimes_{t=1}^{x} q_{st}\right) = \left(\begin{array}{c} \left[1-\prod_{\substack{1\le s_1\le,\dots,\\ <s_x\le m}}\left(1-\prod_{t=1}^{x} a_{st}^{l}\right), 1-\prod_{\substack{1\le s_1\le,\dots,\\ <s_x\le m}}\left(1-\prod_{t=1}^{x} a_{st}^{u}\right)\right], \\ \left[\left(\prod_{\substack{1\le s_1\le\cdots\\ <s_x m}} 1-\left(\prod_{t=1}^{x}(1-b_{st}^{l})\right)\right), \left(\prod_{\substack{1\le s_1\le\cdots\\ <s_x\le m}} 1-\left(\prod_{t=1}^{x}(1-b_{st}^{u})\right)\right)\right], \\ \left[\left(\prod_{\substack{1\le s_1\le\cdots\\ <s_x\le m}} 1-\left(\prod_{t=1}^{x}(1-d_{st}^{l})\right)\right), \left(\prod_{\substack{1\le s_1\le\cdots\\ <s_x\le m}} 1-\left(\prod_{t=1}^{x}(1-d_{st}^{u})\right)\right)\right] \end{array} \right)$$

$$\frac{1}{C_m^x} \bigoplus_{\substack{1\le s_1\le\cdots\\ <s_x\le m}} \left(\bigotimes_{t=1}^{x} q_{st}\right)$$

$$= \left(\begin{array}{c} \left[1-\left(\prod_{\substack{1\le s_1\le\cdots\\ <s_x\le m}}\left(1-\prod_{t=1}^{x} a_{st}^{l}\right)\right)^{1/C_m^x}, 1-\left(\prod_{\substack{1\le s_1\le\cdots\\ <s_x\le m}}\left(1-\prod_{t=1}^{x} a_{st}^{u}\right)\right)^{1/C_m^x}\right], \\ \left[1-\left(\prod_{\substack{1\le s_1\le\cdots\\ <s_x\le m}}\left(1-\prod_{t=1}^{x}(1-b_{st}^{l})\right)\right)^{1/C_m^x}, 1-\left(\prod_{\substack{1\le s_1\le\cdots\\ <s_x\le m}}\left(1-\prod_{t=1}^{x}(1-b_{st}^{u})\right)\right)^{1/C_m^x}\right] \\ \left[1-\left(\prod_{\substack{1\le s_1\le\cdots\\ <s_x\le m}}\left(1-\prod_{t=1}^{x}(1-d_{st}^{l})\right)\right)^{1/C_m^x}, 1-\left(\prod_{\substack{1\le s_1\le\cdots\\ <s_x\le m}}\left(1-\prod_{t=1}^{x}(1-d_{st}^{u})\right)\right)^{1/C_m^x}\right] \end{array} \right) \quad (3)$$

Therefore,

$$IVPFFMSM(q_1, q_2, \dots, q_m)$$
$$= \left(\left[\begin{array}{c} \left(1 - \left(\prod_{\substack{1 \le s_1 \le \cdots \\ < s_x \le m}} \left(1 - \left(\prod_{t=1}^{x} a_{st}^{l}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}, \\ \left(1 - \left(\prod_{\substack{1 \le s_1 \le \cdots \\ < s_x \le m}} \left(1 - \left(\prod_{t=1}^{x} a_{st}^{u}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}} \end{array} \right], \left[\begin{array}{c} 1 - \left(1 - \left(\prod_{\substack{1 \le s_1 \le \cdots \\ < s_x \le m}} \left(1 - \left(\prod_{t=1}^{x} (1 - b_{st}^{l})\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}, \\ 1 - \left(1 - \left(\prod_{\substack{1 \le s_1 \le \cdots \\ < s_x \le m}} \left(1 - \left(\prod_{t=1}^{x} (1 - b_{st}^{u})\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}} \end{array} \right], \\ \left[\begin{array}{c} 1 - \left(1 - \left(\prod_{\substack{1 \le s_1 \le \cdots \\ < s_x \le m}} \left(1 - \left(\prod_{t=1}^{x} (1 - d_{st}^{l})\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}, \\ 1 - \left(1 - \left(\prod_{\substack{1 \le s_1 \le \cdots \\ < s_x \le m}} \left(1 - \left(\prod_{t=1}^{x} (1 - d_{st}^{u})\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}} \end{array} \right] \right)$$

The above-defined IVPFMSM operators fulfilled the criterion of aggregation given as follows.

Property 1: (Idempotency property)

Let $q_{st} = ([a_{st}^l, a_{st}^u], [b_{st}^l, b_{st}^u], [d_{st}^l, d_{st}^u])$ be a collection of IVPFMSMNs. If $q_{st} = q$ then (all are identical)

$$IVPFMSM(q_1, q_2, q_3, \dots, q_m) = q$$

Property 2: (Boundedness Property)

Let $q_{st} = ([a_{st}^l, a_{st}^u], [b_{st}^l, b_{st}^u], [d_{st}^l, d_{st}^u])$ be a collection of IVPFMSMNs such that

$$q^- = \min(q_{st}) = ([min(a_{st}^l), min(a_{st}^u)], [max(b_{st}^l), max(b_{st}^u)], [max(d_{st}^l), max(d_{st}^u)])$$

and

$$q^+ = max(q_{st}) = ([max(a_{st}^l), max(a_{st}^u)], [min(b_{st}^l), min(b_{st}^u)], [min(d_{st}^l), min(d_{st}^u)])$$

Then

$$q^- \le IVPFFMSM(q_1, q_2, \dots, q_m) \le q^+$$

Property 3: (Monotonicity property) Let q_{st} and $\breve{q}_{st}$ be two IVPFMSMNs. If $a_{st}^l \ge \breve{a}_{st}^l$, $a_{st}^u \ge \breve{a}_{st}^u$, $b_{st}^l \ge \breve{b}_{st}^l$, $b_{st}^u \ge \breve{b}_{st}^u$, and $d_{st}^l \ge \breve{d}_{st}^l$, $d_{st}^u \ge \breve{d}_{st}^u$ then

$$IVPFFMSM(q_1, q_2, \dots, q_m) \ge IVPFFMSM(\breve{q}_1, \breve{q}_2, \dots, \breve{q}_m)$$

Definition 6: Let $q_{st} = ([a_{st}^l, a_{st}^u], [b_{st}^l, b_{st}^u], [d_{st}^l, d_{st}^u])$ be a collection of IVPFMSMNs. Then, the IVPFWMSM operator is given by

$$IVPFWMSM(q_1, q_2, \dots, q_m) = \left[\left(\frac{\bigoplus_{1 \le s_1 \le \cdots s_x} \left(\bigotimes_{t=1}^{x} q_{st}^{l} \right)^{\omega_{st}}}{C_m^x} \right)^{1/\dot{x}}, \left(\frac{\bigoplus_{1 \le s_1 \le \cdots \le s_x} \left(\bigotimes_{t=1}^{x} q_{st}^{u} \right)^{\omega_{st}}}{C_m^x} \right)^{1/\dot{x}} \right]$$

Theorem 2: Let $q_{st} = ([a_{st}^l, a_{st}^u], [b_{st}^l, b_{st}^u], [d_{st}^l, d_{st}^u])$ be a collection of IVPFMSMNs. Then, using IVPFWMSM operator, we get

IVPF$F\ WMSM(q_1,q_2,\ldots\ldots,q_m)=$

$$\left(\begin{array}{c}\left[\begin{array}{c}\left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{\dot{x}}(a_{st}^{l})^{\omega_{st}}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}},\\ \left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-(a_{st}^{u})\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}\end{array}\right],\left[\begin{array}{c}1-\left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{\dot{x}}(1-b_{st}^{l})^{\omega_{st}}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}},\\ 1-\left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x}(1-b_{st}^{u})^{\omega_{st}}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}\end{array}\right],\\ \left[\begin{array}{c}1-\left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x}(1-d_{st}^{l})^{\omega_{st}}\right)\right)\right)^{1/C_m^x}\right)^{1/x},\\ 1-\left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x}(1-d_{st}^{u})^{\omega_{st}}\right)\right)\right)^{1/C_m^x}\right)^{1/x}\end{array}\right]\end{array}\right)$$

Proof: As we discussed in above (Theorem 01)

3.1. Interval valued PFDMSM Operators

The purpose of this part to made a concept of IVPFDMSM operators and IVPFDWMSM operators using MD, NMD and AD.

Definition 7: Let $q_{st}=([a_{st}^{l},a_{st}^{u}],[b_{st}^{l},b_{st}^{u}],[d_{st}^{l},d_{st}^{u}])$be a collection of IVPFNs. Then, IVPFDMSM is defined as

$$IVPFDMSM(q_1,q_2,\ldots,q_m)=\left[\frac{1}{x}\left(\bigoplus_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(\bigotimes_{t=1}^{x}q_{st}^{l}\right)^{1/C_m^x}\right),\frac{1}{x}\left(\bigoplus_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(\bigotimes_{t=1}^{x}q_{st}^{u}\right)^{1/C_m^x}\right)\right]$$

Theorem 3: Let $q_{st}=([a_{st}^{l},a_{st}^{u}],[b_{st}^{l},b_{st}^{u}],[d_{st}^{l},d_{st}^{u}])$ denote the collection of IVPFNs. Then, by using IVPFDMSM operators, we have

$IVPFDMSM(q_1,q_2,\ldots,q_m)$

$$=\left(\begin{array}{c}\left[\begin{array}{c}1-\left(1-\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\prod_{t=1}^{x}(1-a_{st}^{l})\right)^{1/C_m^x}\right)^{1/\dot{x}},\\ 1-\left(1-\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\prod_{t=1}^{x}(1-a_{st}^{u})\right)^{1/C_m^x}\right)^{1/\dot{x}}\end{array}\right],\left[\begin{array}{c}\left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x}b_{st}^{l}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}},\\ \left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x}b_{st}^{u}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}\end{array}\right],\\ \left[\begin{array}{c}\left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x}d_{st}^{l}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}},\\ \left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x}d_{st}^{u}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}\end{array}\right]\end{array}\right)$$

Proof: Using Definition 8, we have

$$\bigotimes_{t=1}^{x}q_{st}=\left(\left[1-\prod_{t=1}^{\dot{x}}(1-a_{st}^{l}),1-\prod_{t=1}^{\dot{x}}(1-a_{st}^{u})\right],\left[\prod_{t=1}^{\dot{x}}b_{st}^{l},\prod_{t=1}^{\dot{x}}b_{st}^{u}\right],\left[\prod_{t=1}^{\dot{x}}d_{st}^{l},\prod_{t=1}^{\dot{x}}d_{st}^{u}\right]\right)$$

$$\left(\bigotimes_{t=1}^{x} q_{st}\right)^{1/C_m^x} = \left(\begin{array}{c} \left[\begin{array}{l} \left(1-\prod_{t=1}^{x}(1-a_{st}^{l})\right)^{1/C_m^x}, \\ \left(1-\prod_{t=1}^{x}(1-a_{st}^{u})\right)^{1/C_m^x} \end{array} \right], \left[\begin{array}{l} 1-\left(1-\left(\prod_{t=1}^{x} b_{st}^{l}\right)\right)^{1/C_m^x}, \\ 1-\left(1-\left(\prod_{t=1}^{x} b_{st}^{u}\right)\right)^{1/C_m^x} \end{array} \right], \\ \left[\begin{array}{l} 1-\left(1-\left(\prod_{t=1}^{x} d_{st}^{l}\right)\right)^{1/C_m^x}, \\ 1-\left(1-\left(\prod_{t=1}^{x} d_{st}^{u}\right)\right)^{1/C_m^x} \end{array} \right] \end{array} \right)$$

$$\dot{\bigotimes}_{\substack{1\le s_1\le\cdots \\ <s_x\le m}} \left(\bigoplus_{t=1}^{x} q_{st}\right)^{1/C_m^x} = \left(\begin{array}{c} \left[\begin{array}{l} \prod_{\substack{1\le s_1\le\cdots \\ <s_x\le m}} \left(1-\prod_{t=1}^{x}(1-a_{st}^{l})\right)^{1/C_m^x}, \\ \prod_{\substack{1\le s_1\le\cdots \\ <s_x\le m}} \left(1-\prod_{t=1}^{x}(1-a_{st}^{u})\right)^{1/C_m^x} \end{array} \right], \left[\begin{array}{l} 1-\prod_{\substack{1\le s_1\le\cdots \\ <s_x\le m}} \left(1-\left(\prod_{t=1}^{x} b_{st}^{l}\right)\right)^{1/C_m^x}, \\ 1-\prod_{\substack{1\le s_1\le\cdots \\ <s_x\le m}} \left(1-\left(\prod_{t=1}^{x} b_{st}^{u}\right)\right)^{1/C_m^x} \end{array} \right], \\ \left[\begin{array}{l} 1-\prod_{\substack{1\le s_1\le\cdots \\ <s_x\le m}} \left(1-\left(\prod_{t=1}^{x} d_{st}^{l}\right)\right)^{1/C_m^x}, \\ 1-\prod_{\substack{1\le s_1\le\cdots \\ <s_x\le m}} \left(1-\left(\prod_{t=1}^{x} d_{st}^{u}\right)\right)^{1/C_m^x} \end{array} \right] \end{array} \right)$$

$$IVPFDMSM(q_1, q_2, \ldots, q_m)$$

$$= \left(\begin{array}{c} \left[\begin{array}{l} 1-\left(1-\prod_{\substack{1\le s_1\le\cdots \\ <s_x\le m}} \left(1-\prod_{t=1}^{x}(1-a_{st}^{l})\right)^{1/C_m^x}\right)^{1/\dot{x}}, \\ 1-\left(1-\prod_{\substack{1\le s_1\le\cdots \\ <s_x\le m}} \left(1-\prod_{t=1}^{x}(1-a_{st}^{u})\right)^{1/C_m^x}\right)^{1/\dot{x}} \end{array} \right], \left[\begin{array}{l} \left(1-\left(\prod_{\substack{1\le s_1\le\cdots \\ <s_x\le m}} \left(1-\left(\prod_{t=1}^{x} b_{st}^{l}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}, \\ \left(1-\left(\prod_{\substack{1\le s_1\le\cdots \\ <s_x\le m}} \left(1-\left(\prod_{t=1}^{x} b_{st}^{u}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}} \end{array} \right], \\ \left[\begin{array}{l} \left(1-\left(\prod_{\substack{1\le s_1\le\cdots \\ <s_x\le m}} \left(1-\left(\prod_{t=1}^{x} d_{st}^{l}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}, \\ \left(1-\left(\prod_{\substack{1\le s_1\le\cdots \\ <s_x\le m}} \left(1-\left(\prod_{t=1}^{x} d_{st}^{u}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}} \end{array} \right] \end{array} \right)$$

As stated above IVPFDMSM operator fulfilled the criterion of aggregation given as follows.

Property 4: (Idempotency property)

Let $q_{st} = ([a_{st}^{l}, a_{st}^{u}], [b_{st}^{l}, b_{st}^{u}], [d_{st}^{l}, d_{st}^{u}])$ be a collection of IVPFMSMNs. If $q_{st} = q$ then (all are identical)

$$IVPFDMSM(q_1, q_2, q_3, \dots, q_m) = q$$

Property 5: (Boundedness property) Let $q_{st} = ([a_{st}^l, a_{st}^u], [b_{st}^l, b_{st}^u], [d_{st}^l, d_{st}^u])$ be a collection of IVPFMSMNs such that

$$q^- = \min(q_{st}) = ([min(a_{st}^l), min(a_{st}^u)], [max(b_{st}^l), max(b_{st}^u)], [max(d_{st}^l), max(d_{st}^u)])$$

and

$$q^+ = max(q_{st}) = ([max(a_{st}^l), max(a_{st}^u)], [min(b_{st}^l), min(b_{st}^u)], [min(d_{st}^l), min(d_{st}^u)])$$

Then

$$q^- \leq IVPFDMSM(q_1, q_2, \dots, q_m) \leq q^+$$

Property 6: (Monotonicity property) Let q_{st} and $\breve{q}_{st}$ be two IVPFMSMNs If $a_{st}^l \geq \breve{a}_{st}^l$ $a_{st}^u \geq \breve{a}_{st}^u$ $b_{st}^l \geq \breve{b}_{st}^l$ $b_{st}^u \geq \breve{b}_{st}^u$, and $d_{st}^l \geq \breve{d}_{st}^l$, $d_{st}^u \geq \breve{d}_{st}^u$ then

$$IVPFDMSM(q_1, q_2, \dots, q_m) \geq IVPFDMSM(\breve{q}_1, \breve{q}_2, \dots, \breve{q}_m)$$

Definition 8: Let $q_{st} = ([a_{st}^l, a_{st}^u], [b_{st}^l, b_{st}^u], [d_{st}^l, d_{st}^u])$ be a collection of IVPDFMSMNs. Then, the IVPFDWMSM operator is given by

$$IVPFDMSM(q_1, q_2, \dots, q_m) = \left[\frac{1}{x}\left(\bigoplus_{\substack{1\leq s_1\leq\cdots\\<s_x\leq m}}\left(\bigotimes_{t=1}^{K}(\hat{w}_{st}\otimes q_{st}^l)\right)^{1/C_m^x}\right), \frac{1}{x}\left(\bigoplus_{\substack{1\leq s_1\leq\cdots\\<s_x\leq m}}\left(\bigotimes_{t=1}^{K}(\hat{w}_{st}\otimes q_{st}^u)\right)^{1/C_m^x}\right)\right]$$

Theorem 4: Let $q_{st} = ([a_{st}^l, a_{st}^u], [b_{st}^l, b_{st}^u], [d_{st}^l, d_{st}^u])$ denote the collection of IVPFNs. Then, by using IVPFDWMSM operators, we have

$IVPFDWMSM(q_1, q_2, \dots, q_m)$

$$= \left(\begin{array}{c}\left[\begin{array}{c}1-\left(1-\prod\limits_{\substack{1\leq s_1\leq\cdots\\<s_x\leq m}}\left(1-\prod\limits_{t=1}^{x}(1-a_{st}^l)^{\omega_{st}}\right)^{1/C_m^x}\right)^{1/\dot{x}},\\ 1-\left(1-\prod\limits_{\substack{1\leq s_1\leq\cdots\\<s_x\leq m}}\left(1-\prod\limits_{t=1}^{x}(1-a_{st}^u)^{\omega_{st}}\right)^{1/C_m^x}\right)^{1/\dot{x}}\end{array}\right], \left[\begin{array}{c}\left(1-\left(\prod\limits_{\substack{1\leq s_1\leq\cdots\\<s_x\leq m}}\left(1-\left(\prod\limits_{t=1}^{x}(b_{st}^l)^{\omega_{st}}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}},\\ \left(1-\left(\prod\limits_{\substack{1\leq s_1\leq\cdots\\<s_x\leq m}}\left(1-\left(\prod\limits_{t=1}^{x}(b_{st}^u)^{\omega_{st}}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}\end{array}\right],\\ \left[\begin{array}{c}\left(1-\left(\prod\limits_{\substack{1\leq s_1\leq\cdots\\<s_x\leq m}}\left(1-\left(\prod\limits_{t=1}^{x}(d_{st}^l)^{\omega_{st}}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}},\\ \left(1-\left(\prod\limits_{\substack{1\leq s_1\leq\cdots\\<s_x\leq m}}\left(1-\left(\prod\limits_{t=1}^{x}(d_{st}^u)^{\omega_{st}}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}\end{array}\right]\end{array}\right)$$

Proof: As we discussed in above (Theorem 03)

4. Consequences of the IVPFMSM operator

In this section, we studied the relationship of the IVPFMSM with other MSM operators.

$$IVPFMSM(q_1, q_2, \ldots\ldots, q_m) = \left(\begin{array}{c} \left[\begin{array}{c} \left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x} a_{st}^{l}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}, \\ \left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x} a_{st}^{u}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}} \end{array} \right], \left[\begin{array}{c} 1-\left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x}(1-b_{st}^{l})\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}, \\ 1-\left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x}(1-b_{st}^{u})\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}} \end{array} \right], \\ \left[\begin{array}{c} 1-\left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x}(1-d_{st}^{l})\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}, \\ 1-\left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x}(1-d_{st}^{u})\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}} \end{array} \right] \end{array} \right)$$

$$IVPFDMSM(q_1, q_2, \ldots, q_m) = \left(\begin{array}{c} \left[\begin{array}{c} 1-\left(1-\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\prod_{t=1}^{x}(1-a_{st}^{l})\right)^{1/C_m^x}\right)^{1/\dot{x}}, \\ 1-\left(1-\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\prod_{t=1}^{x}(1-a_{st}^{u})\right)^{1/C_m^x}\right)^{1/\dot{x}} \end{array} \right], \left[\begin{array}{c} \left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x} b_{st}^{l}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}, \\ \left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x} b_{st}^{u}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}} \end{array} \right], \\ \left[\begin{array}{c} \left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x} d_{st}^{l}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}}, \\ \left(1-\left(\prod_{\substack{1\le s_1\le\cdots\\<s_x\le m}}\left(1-\left(\prod_{t=1}^{x} d_{st}^{u}\right)\right)\right)^{1/C_m^x}\right)^{1/\dot{x}} \end{array} \right] \end{array} \right)$$

For $b_{st} = 0$ the IVPFMSM and IVPFDMSM reduced into the MSM operators of IVIFSs, given as follows:

$$IVIFMSM(q_1, q_2, \ldots.., q_m) = \left(\begin{array}{c} \left[\begin{array}{c} \left(1 - \left(\prod_{\substack{1 \le s_1 \le \cdots \\ < s_x \le m}} \left(1 - \left(\prod_{t=1}^{x} a_{st}^{l} \right) \right) \right)^{1/C_m^x} \right)^{1/\dot{x}}, \\ \left(1 - \left(\prod_{\substack{1 \le s_1 \le \cdots \\ < s_x \le m}} \left(1 - \left(\prod_{t=1}^{x} a_{st}^{u} \right) \right) \right)^{1/C_m^x} \right)^{1/\dot{x}} \end{array} \right], \\ \left[\begin{array}{c} 1 - \left(1 - \left(\prod_{\substack{1 \le s_1 \le \cdots \\ < s_x \le m}} \left(1 - \left(\prod_{t=1}^{x} (1 - d_{st}^{l}) \right) \right) \right)^{1/C_m^x} \right)^{1/\dot{x}}, \\ 1 - \left(1 - \left(\prod_{\substack{1 \le s_1 \le \cdots \\ < s_x \le m}} \left(1 - \left(\prod_{t=1}^{x} (1 - d_{st}^{u}) \right) \right) \right)^{1/C_m^x} \right)^{1/\dot{x}} \end{array} \right] \end{array} \right)$$

$$IVIFDMSM(q_1, q_2, \ldots, q_m) = \left(\begin{array}{c} \left[\begin{array}{c} 1 - \left(1 - \prod_{\substack{1 \le s_1 \le \cdots \\ < s_x \le m}} \left(1 - \prod_{t=1}^{x} (1 - a_{st}^{l}) \right)^{1/C_m^x} \right)^{1/\dot{x}}, \\ 1 - \left(1 - \prod_{\substack{1 \le s_1 \le \cdots \\ < s_x \le m}} \left(1 - \prod_{t=1}^{x} (1 - a_{st}^{u}) \right)^{1/C_m^x} \right)^{1/\dot{x}} \end{array} \right], \\ \left[\begin{array}{c} \left(1 - \left(\prod_{\substack{1 \le s_1 \le \cdots \\ < s_x \le m}} \left(1 - \left(\prod_{t=1}^{x} d_{st}^{l} \right) \right) \right)^{1/C_m^x} \right)^{1/\dot{x}}, \\ \left(1 - \left(\prod_{\substack{1 \le s_1 \le \cdots \\ < s_x \le m}} \left(1 - \left(\prod_{t=1}^{x} d_{st}^{u} \right) \right) \right)^{1/C_m^x} \right)^{1/\dot{K}} \end{array} \right] \end{array} \right)$$

5. An approach to IVPFMSM in MADM process

As we know, multi attribute decision making process (MADM) is a best way to select a better option in daily life problems. In this part of paper, we studied MADM process using IVPFMSM and IVPFDMSM operators to aggregate the results. We also find the aggregated results of IVPFWMSM and IVPFDWMSM operators to choose best one. As we know that MADM process consist on a set of attributes and set of alternatives. Let a set of attributes $V = \{\ddot{V}_1, \ddot{V}_2, \ldots, \ddot{V}_m\}$ under observation and let $Z = \{\ddot{Z}_1, \ddot{Z}_2, \ldots, \ddot{Z}_m\}$ be a set of alternatives. The undetermined data is based on IVPFNs where the four side of specialist options are considered to analyze the alternatives using IVPFNs. First, we aggregate the results of IVPFMSM and IVPFDMSM operators for the chosen of best option. The algorithm includes the following steps.

Step 1. The specialist using IVPFNs in the form of MD, NMD, AD and RD under restrains of IVPFNs. The performance of alternatives $V_s \in V$ with respect to attributes$Z_t \in Z$ is expressed by an IVPFN $q_{st} = ([a_{st}^l, a_{st}^u], [b_{st}^l, b_{st}^u], [d_{st}^l, d_{st}^u])$ such that $0 \le ([a_{st}^u + b_{st}^u + d_{st}^u]) \le 1$.

Step 2. Generally, attributes are divided into two types, first is benefit attribute and second is cost attributes. To deal with cost factor, we used procedure of normalization in which every cost attribute is change into benefit attribute.

$$q_{st} = ([a_{st}^l, a_{st}^u], [b_{st}^l, b_{st}^u], [d_{st}^l, d_{st}^u]) = \begin{Bmatrix} ([a_{st}^l, a_{st}^u], [b_{st}^l, b_{st}^u], [d_{st}^l, d_{st}^u]), & \text{benefit type of attribute} \\ ([d_{st}^l, d_{st}^u], [b_{st}^l, b_{st}^u], [a_{st}^l, a_{st}^u]), & cost\ type\ o\ f\ attribute \end{Bmatrix}$$

Step 3. After applying normalization process, we utilize two aggregation operators IVPFMSM and IVPFDMSM.

Step 4. After aggregate the values of IVPFNs data, we use score function to arrange the obtained information.

Step 5. Finally, we obtained ranking of score function (previous step).

Information science is a demanding task that has been talk over widely by different authors. In this research, we suppose the Company Benefit Plan (CBP) problem in which specialist chose a best option for the benefit of a company. Now an elucidative example for MADM is provided as an application based on the proposed IVPFMSM, IVWPFMSM, IVPFDMSM and IVPFWMSM operators.

Example 1

Consider a set of alternatives in CBP schemes $\{\ddot{Z}_1, \ddot{Z}_2, \ddot{Z}_3, \ddot{Z}_4\}$ that need to be estimation by specialists. The CBP scheme has a set of finite attributes are denoted $\{\ddot{V}_1, \ddot{V}_2, \ddot{V}_3, \ddot{V}_4\}$. The aim is to select the best option CBP schemes for company. In which Z_1 represents stock purchases; z_2 represents stock awards; Z_3 represents change of control, and Z_4 represents bonus of the company. Each attribute is associated by a weight given the weight vector $w_{st} = (0.30, 0.30, 0.20, 0.20)^T$. The procedure of the CBP scheme is elaborated as follows.

Table 1. Evaluation remarks about CBP schemes based on IVPFNs.

			V1						**V2**			
	al	Au	Bl	Bu	dl	du	Al	au	bl	Bu	dl	du
$\mathbf{Z_1}$	0.2	0.5	0.1	0.3	0.1	0.2	0.1	0.4	0.1	0.2	0.1	0.2
$\ddot{\mathbf{Z}}_2$	0.1	0.3	0.1	0.2	0.1	0.3	0.2	0.3	0.1	0.4	0.2	0.3
$\ddot{\mathbf{Z}}_3$	0.2	0.4	0.2	0.4	0.2	0.3	0.1	0.2	0.3	0.4	0.1	0.2
$\ddot{\mathbf{Z}}_4$	0.1	0.2	0.1	0.3	0.1	0.2	0.1	0.3	0.2	0.3	0.3	0.4
			V3						V4			
	Al	Au	Bl	Bu	dl	du	Al	au	Bl	Bu	Dl	du
$\ddot{\mathbf{Z}}_1$	0.2	0.3	0.1	0.2	0.2	0.3	0.1	0.2	0.1	0.3	0.1	0.2
$\ddot{\mathbf{Z}}_2$	0.1	0.2	0.2	0.3	0.1	0.2	0.2	0.3	0.2	0.4	0.2	0.3
$\ddot{\mathbf{Z}}_3$	0.2	0.4	0.3	0.4	0.2	0.3	0.3	0.4	0.2	0.3	0.3	0.4
$\ddot{\mathbf{Z}}_4$	0.1	0.3	0.1	0.3	0.1	0.3	0.1	0.3	0.1	0.2	0.1	0.4

Table 2. Aggregated results using IVPFMSM operators

			V1						**V2**			
	Al	Au	Bl	Bu	Dl	du	Al	Au	bl	Bu	dl	Du
$\ddot{\mathbf{Z}}_1$	0.14	0.33	0.12	0.30	0.12	0.25	0.12	0.29	0.17	0.32	0.17	0.27
$\ddot{\mathbf{Z}}_2$	0.91	0.91	0.02	0.05	0.09	0.04	0.91	0.91	0.03	0.06	0.03	0.05
$\ddot{\mathbf{Z}}_3$	0.15	0.35	0.12	0.29	0.12	0.24	0.12	0.30	0.16	0.31	0.16	0.26
$\ddot{\mathbf{Z}}_4$	0.03	0.06	0.91	0.91	0.91	0.91	0.02	0.05	0.91	0.91	0.91	0.91
			V3						V4			
	Al	Au	Bl	Bu	Dl	du	al	Au	bl	Bu	Dl	Du
$\ddot{\mathbf{Z}}_1$	0.14	0.29	0.17	0.30	0.15	0.27	0.16	0.29	0.15	0.30	0.17	0.32
$\ddot{\mathbf{Z}}_2$	0.91	0.91	0.03	0.05	0.03	0.05	0.91	0.91	0.03	0.05	0.03	0.06
$\ddot{\mathbf{Z}}_3$	0.15	0.30	0.16	0.29	0.14	0.27	0.17	0.30	0.14	0.29	0.16	0.31
$\ddot{\mathbf{Z}}_4$	0.03	0.05	0.91	0.91	0.91	0.91	0.03	0.05	0.91	0.91	0.91	0.91

Table 3. The score values of IVPFNs, obtained in Table 2.

Score	Z1	Z2	Z3	Z4
IVPFMSM	0.0865	0.0648	0.0739	0.0728
IVPFWMSM	0.5373	0.5506	0.5532	0.5511
IVPFDMSM	0.0923	0.0699	0.0783	0.0797
IVPFDWMSM	-0.0265	-0.0233	-0.0246	-0.0259

Step 1. The managements of the company examine the initially values of CBP and gives their opinions using IVPFNs.

Step 2. Normalization process is not required because all attributes are benefit type.

Step 3. In this step, we get aggregation results based on IVPNs using IVPFMSM, IVPFWMSM, IVPFDMSM and IVPFDMSM operators in table 2.

Step 4. we Apply definition of score function on the aggregated values and make ranking of the results obtained in table 3. We can see the ranking in table 4 and in Figure 1.

Step 5. The end

Table 4. Ranking of CBP schemes

Method	Ranking values
IVPFMSM	$\ddot{Z}_1 > \ddot{Z}_3 > \ddot{Z}_4 > \ddot{Z}_2$
IVPFWMSM	$\ddot{Z}_1 > \ddot{Z}_3 > \ddot{Z}_4 > \ddot{Z}_2$
IVPFDMSM	$\ddot{Z}_1 > \ddot{Z}_4 > \ddot{Z}_3 > \ddot{Z}_2$
IVPFDWMSM	$\ddot{Z}_2 > \ddot{Z}_3 > \ddot{Z}_4 > \ddot{Z}_1$

From table 4, it is clear that IVPFMSM, IVPFWMSM, IVPFDMSM give us $\ddot{Z}_1$ As most effective one in CBP scheme while the IVPFDWMSM give us $\ddot{Z}_2$ As best one. We get different results using any MSM operators that consisted on the desire of experts. We can see the ranking results in Figure 1.

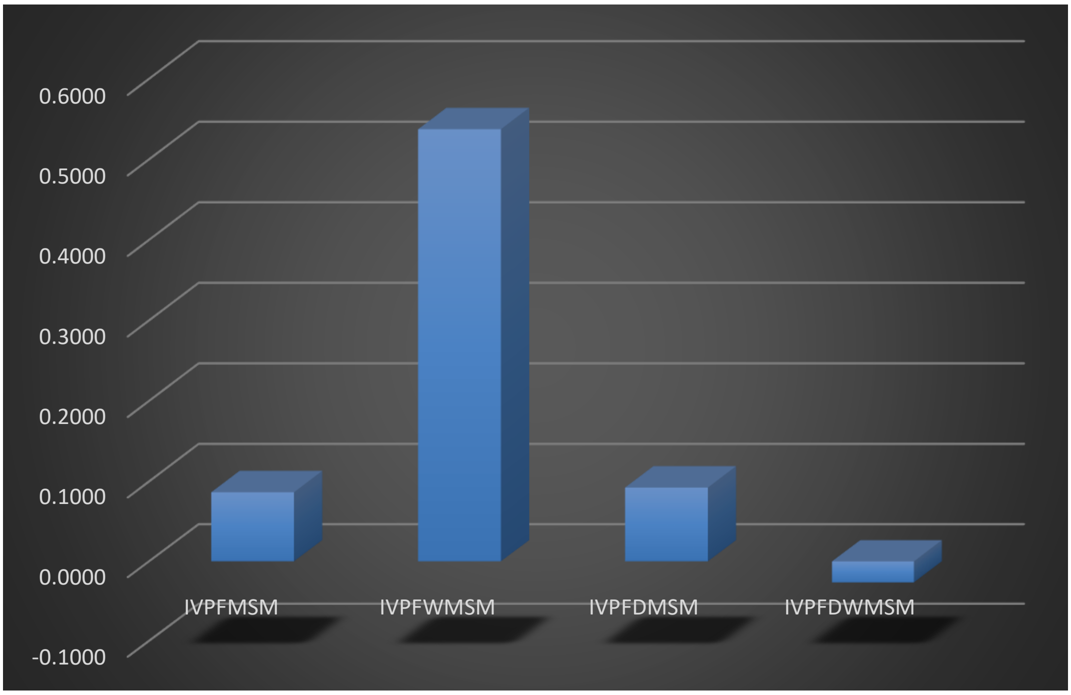

Figure 1. Ranking results

From Figure 1. It is clear that IVPFMSM operator is best one. In CBP scheme, stock purchasing is considered best option for the benefit of company under MADM process.

6. Comparative study

In this section, we will compare MSM operators with other developed AOs of PFNs such as IVPFHWA operators, IVPFHWG operators, IVPFWA operators, IVPFWG operators, IVPFDWA operators and IVPFDWG operators.

A PFS is a structure where we can minimize the loss of uncertain information. In the few years, an exalted number of AOs have been developed. Here, we can see a comparative analysis of proposed operator with other developed AOs.

Table 5. Comparison with existing operators

Operators	$\ddot{A}_1$	$\ddot{A}_2$	$\ddot{A}_3$	$\ddot{A}_4$
IVPFHWA	0.0837	0.0696	0.0679	0.0664
IVPFHWG	0.0978	0.0749	0.0802	0.0769
IVPFWA	0.0943	0.0708	0.0766	0.0754
IVPFWG	0.1013	0.0806	0.0833	0.0848
IVPFDWA	0.0819	0.0637	0.0689	0.0574
IVPFMSM	0.0865	0.0648	0.0739	0.0728
IVPFWMSM	0.5373	0.5506	0.5532	0.5511
IVPFDMSM	0.0923	0.0699	0.0783	0.0797
IVPFWMSM	-0.0265	-0.0233	-0.0246	-0.0259

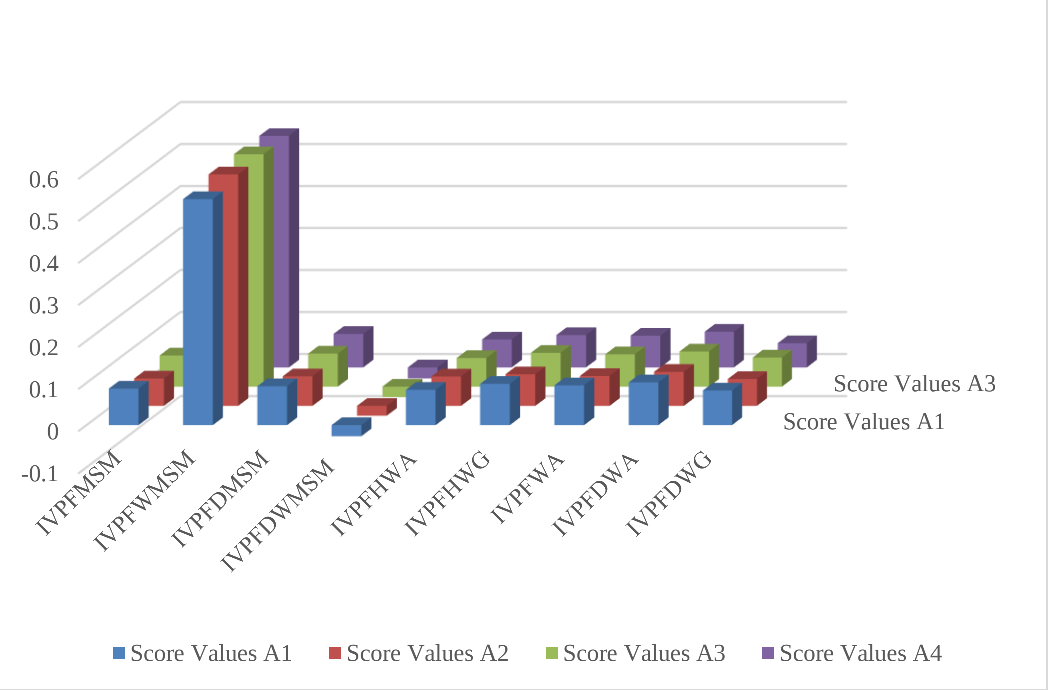

Figure 2. Comparisons results

From Figure 2. we can see that in IVPFMSM operator A_2 is most effective than other, in IVPFWMSM operator A_1 is most effective tools, in IVPFDMSM operator A_2 is best, in IVPFDWMSM operator A_3 is best tool. Similarly, in other operators we can see A_2 is most effective than others.

In comparative analysis we can see the results of IVHAO, IVDAO, IVArAO and IVGAO obtained by MSM operator. All others AOs do not describe the relationship between input Arguments. But newly developed MSM operators describes it. No one else operators such As IFSs (Liu et al., 2020), PyFSs, And qROFSs show such accurate data. This is because it often does not have AD and RD which damages information. For good understanding, we summarized the results in Figure 2.

7. Conclusion

In this article, we developed MSM operators using IVPF data. We get two basic benefits by using IVPFMSM operator. It uses four kinds of degrees, i.e., MD, NMD, AD, and RD to show the undefined data as a specialist opinion for solving MADM problem. The more Advantage of MSM operators is to correlate the input arguments in aggregation process under uncertain condition. So, MSM operators is more important and elastic than traditionally operators such as Arithmetic mean operators, geometric mean operators, Dombi t-norm and t-conorm operators, Hamacher t-norm and t-conorm operators etc. These operators point to likewise two input values, contrasted other traditional aggregation operators. That's why MSM operators are more valid rather than other developed operators. First, we proposed a notion of MSM in the form of IVPFMSM operators on IVPFS and explains three properties of aggregation process (Idempotency, Boundedness, monotonicity). After that we explains the uniqueness of proposed operator (IVPFMSM) and took a comparative analysis of it with other developed operators. We show that IVPFMSM operator is more effective than others operators. At the end, we use process of multi-Attributes decision making (MADM) to show the importance of IVPFMSM operator. We also discuss an example (Company Benefit Plan) mathematically using MADM process. We will be getting this kind of work in future in the shape of IVSFSs, IVTSFSs and IVCTSFs.

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